EXERCISES FOR INF3320

LINEAR, AFFINE, AND PROJECTIVE TRANSFORMS (PART I)

14/09/202

1. A point $\mathbf{p} = [x, y]$ in $\mathbb{R}^2$ can be represented in polar coordinates $[\rho, \phi]$ with

$$x = \rho \cos \phi, \qquad y = \rho \sin \phi.$$

   (a) What is the new position $\mathbf{p}' = [x', y']$ after a rotation of θ about the origin?

   (b) Write down a 2×2 matrix for the transformation from $\mathbf{p}$ to $\mathbf{p}'$, i.e. find $\mathbf{M}$ such that

$$\mathbf{p}' = \mathbf{M}\mathbf{p}.$$

   (c) Use the solution of the previous exercise to find a 4×4 rotation matrix that represent a rotation of θ in $\mathbb{R}^3$ about the y-axis.

2. Let $\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_n$ be a sequence of transformations, where each $\mathbf{M}_i$ is either a rotation around the z-axis or an arbitrary translation in $\mathbb{R}^3$.

   Show that

$$\mathbf{M}_1\mathbf{M}_2 \cdots \mathbf{M}_n = \mathbf{T}\mathbf{R}$$

   where $\mathbf{T}$ is a translation and $\mathbf{R}$ is a rotation around the z-axis.

3. (a) Write down a matrix $\mathbf{M}$ that mirrors points about the xy-plane.

   Given an angle θ and a normalized axis of rotation $\mathbf{a}$, let $\mathbf{R}_{\mathbf{a},\theta}$ be the corresponding rotation matrix. Given a translation vector $\mathbf{t}$, let $\mathbf{T}_{\mathbf{t}}$ be the corresponding translation matrix. Using $\mathbf{M}$, $\mathbf{R}_{\mathbf{a},\theta}$ and $\mathbf{T}_{\mathbf{t}}$,

   (b) specify a 4×4-matrix that mirrors points about a plane through the origin, and then

   (c) specify a 4×4-matrix that mirrors points about any plane.

4. Let $\mathbf{p}$ be a point in $\mathbb{R}^3$, $\mathbf{n}$ a surface normal, and $\mathbf{M}$ a 4×4 non-singular homogenous transformation matrix.

   Using a right hand coordinate system (as OpenGL do):

   (a) How do we apply $\mathbf{M}$ on $\mathbf{p}$ to find the transformed point $\mathbf{p}'$?

   (b) How do we apply $\mathbf{M}$ on $\mathbf{n}$ to find the transformed surface normal $\mathbf{n}'$? Why?